Spring 2020: Mechanical Properties Materials Science (MPMS)

Take Home Exam I: (Elasticity)
Given out on Wednesday 02/19/2020, due in class on Wednesday 02/26/2020

Each question carries equal points; Please give your answers on the enclosed pages.

This exam is not very long. May I therefore ask you to think very carefully about your answers, and write them down in clear English. If you include derivations, then explain each step, and state the assumptions in the analysis.

The grade will depend on the clarity of your presentation.

Below “very short” answers are enclosed. (03/04/2020)

1. Give three examples of iconic figures, or equations that pertain to the understanding of elastic deformation in solids.

(i) The Youngs Modulus

(ii) The Poisson's Ratio

(iii) Elastic energy per unit volume

(iv) Units of stress are energy per unit volume

2. Give a definition of the Youngs Modulus, and describe a cantilever beam experiment to measure it. Why is this method better than a simple uniaxial experiment.

The Youngs Modulus is the linear portion of the early slope of the stress – strain curve obtained under uniaxial tension.

The cantilever beam can have much lower stiffness than a uniaxial test bar. Furthermore the stiffness can be "designed" by changing the thickness and the length of the beam. A lower stiffness reduces the error in the measurement of the modulus.

3. Assuming a sinusoidal force displacement curve between atoms separated by a distance $\Omega^{1/3}$, where Omega is the volume per atom, derive the following equation that relates the elastic modulus to the heat of melting.

$$E^* = \frac{\Delta H_V}{\Omega}\frac{8\pi^2}{3}$$

In class we discovered that the above equation underestimated the elastic modulus of a metal, say copper. Sketch a modification to the force displacement curve, relative to the sinusoidal form, that can bring a closer agreement between theory and experiment.

In the derivation be careful to point out the maximum in the force (that is the fracture point) often occurs at 15% of the elastic strain (from experiment) in ideal materials such as highly polished optical glass fibers.

For the second part draw the curve with a higher initial slope but have it decline quickly so that the area under it and under the sinusoidal curve remain the same.

4. Write short answers to the following statements:

(i) Covalent bonds are usually stronger than metallic bonds.

Covalent bonds are local and directional allowing the electron lobes to overlap so that the electrons can be shared among nearest neighbors. In metallic bonding the electrons are shared more generally with many atoms thereby weakening the bonds.

(ii) Compounds made to constituents with larger differences between their electronegativity values, are more likely to be ionically bonded.

The electronegativity of an atom relates to its affinity for electrons. Thus an atom with lower electronegativity has a weaker affinity to electrons than the atom with a higher electronegativity. In and ideal ionic bond one atom gives up an electron with the other accepts it, creating an electrostatic force between the atoms (incidentally this force is non-directional).

(iii) Consider the Youngs Modulus and the Poisson's ratio at the two independent elastic constants in elastic deformation of an isotropic solid. *Explain in words* how the Bulk modulus may be related to the elastic modulus and the Poisson's ratio. The equation is given just below.

$$B = \frac{E}{3(1-2\nu)}$$

Bulk modulus is defined under a hydrostatic pressure or tension. Youngs modulus and the Poisson's ratio refer to a uniaxial stress. A higher Poisson's ratio means there is greater contraction in the transverse direction. The resistance to this contraction is greater in a hydrostatic tension experiment, therefore a higher value of the Poisson's ratio will increase the hydrostatic tension required to preclude transverse strain.

5. Using the equations provided in the notes for the elastic deformation of a spherical inclusion embedded within a matrix, show that incompatible expansion of the two materials will create only a shear stress in the surrounding matrix.

The requirement for pure shear deformation is that the sum of the three principal stresses must be equal to zero, that is there is no hydrostatic tension. In the current problem the principal stresses are

$$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\theta\theta}$$

The equations show that $\sigma_{rr}, = -2\sigma_{\theta\theta}$ hence pure shear.